AGA0106
Astronomia de Posição

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Triângulo de posição

Agradecimentos: Prof. Roberto Boczko e Prof. Alan Alves Brito

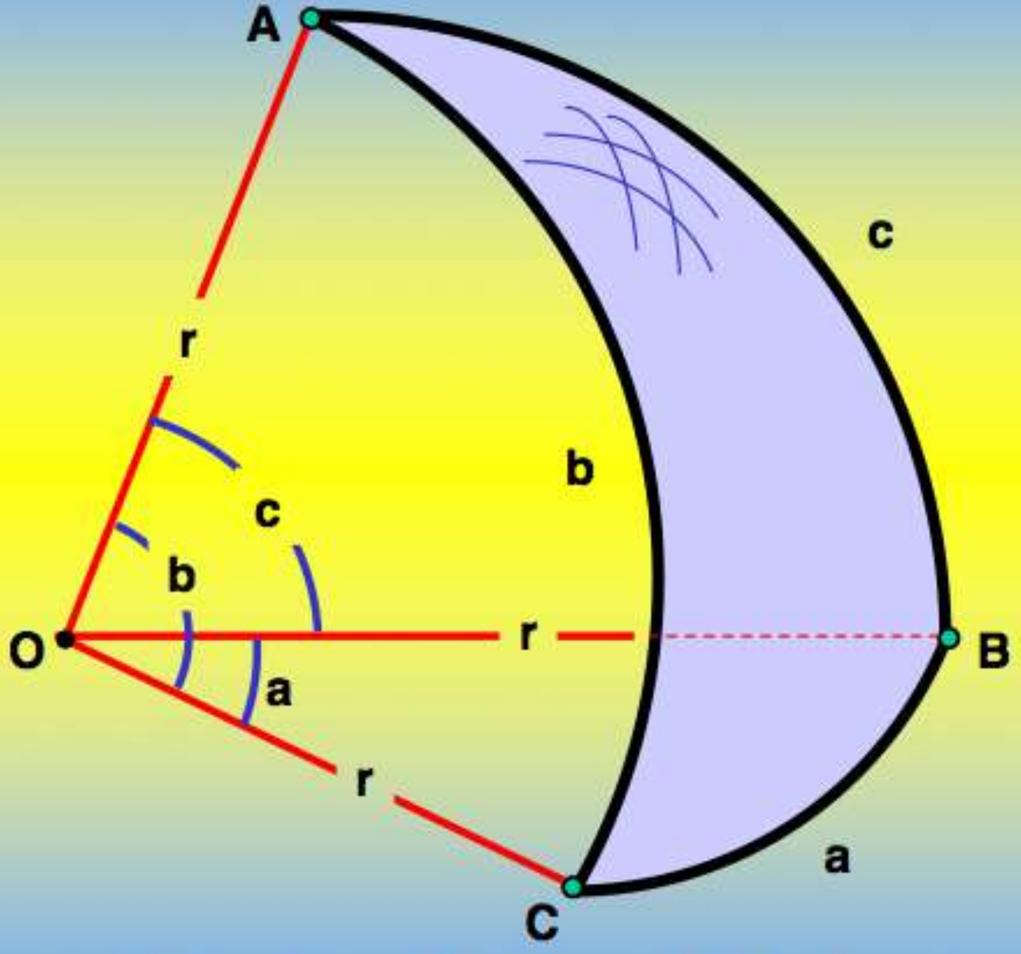
Aula A08

Tópicos da aula

- Trigonometria esférica: revisão;
- Triângulo de posição (estudo analítico do movimento diurno);
- Culminações;
- Nascer e ocaso;
- Cruzamento com o primeiro e segundo verticais.

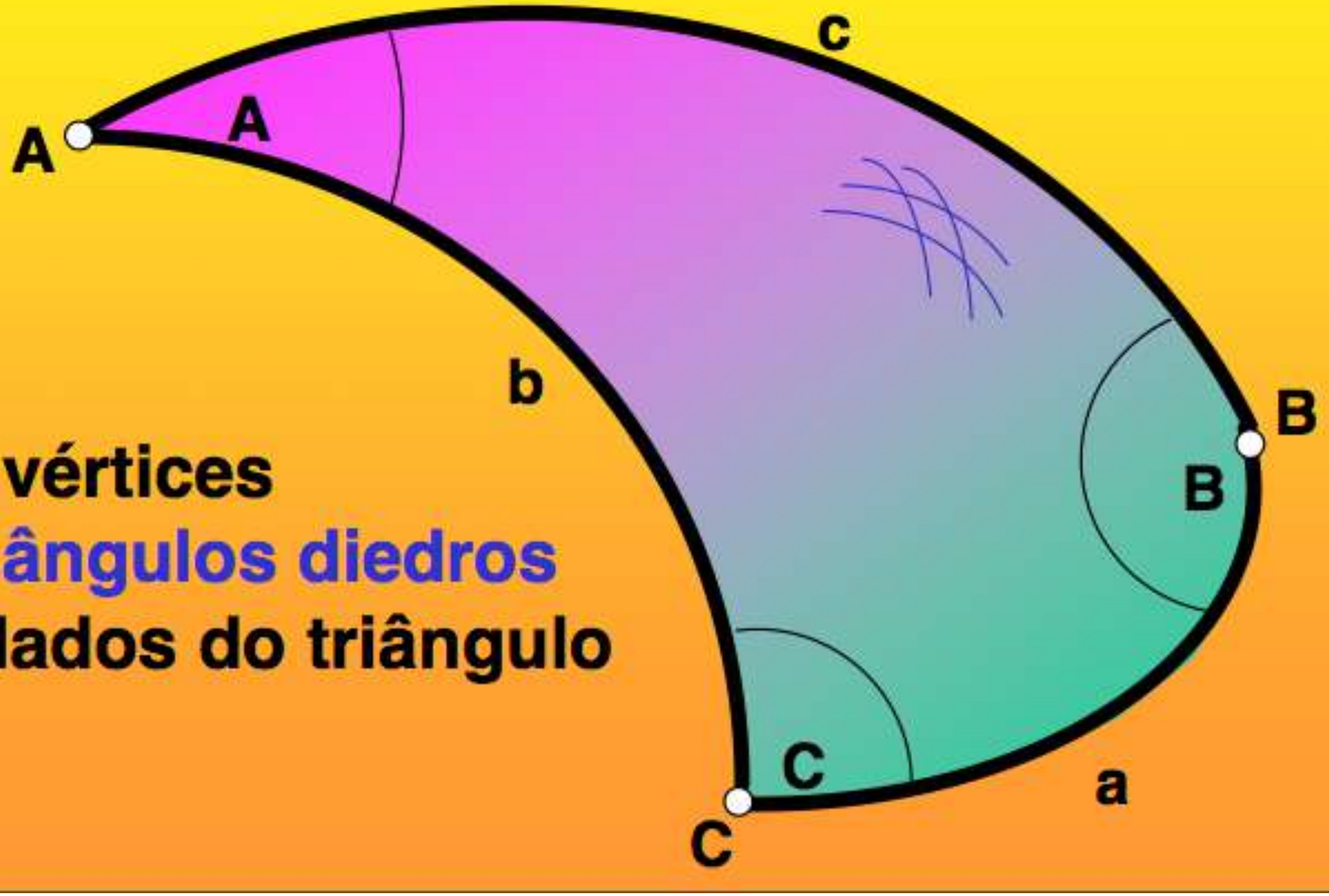
Trigonometrica esférica: brevíssima revisão

Lados do triângulo esférico



a , b , c : lados do triângulo esférico = medidas dos ângulos centrais

Elementos de um Triângulo Esférico

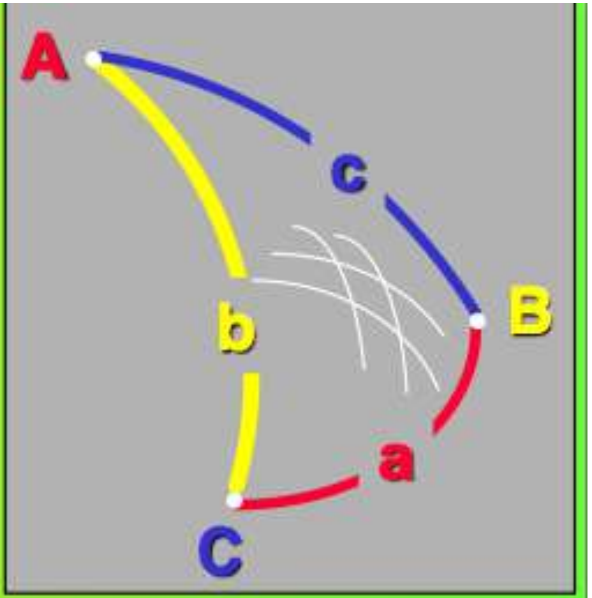


A,B,C = vértices

A,B,C = ângulos diedros

a,b,c = lados do triângulo

Resumo das Fórmulas de Trigonometria Esférica



Co-seno

$$\cos a = \cos b \,.\, \cos c + \operatorname{sen} b \,.\, \operatorname{sen} c \,.\, \cos A$$

Seno

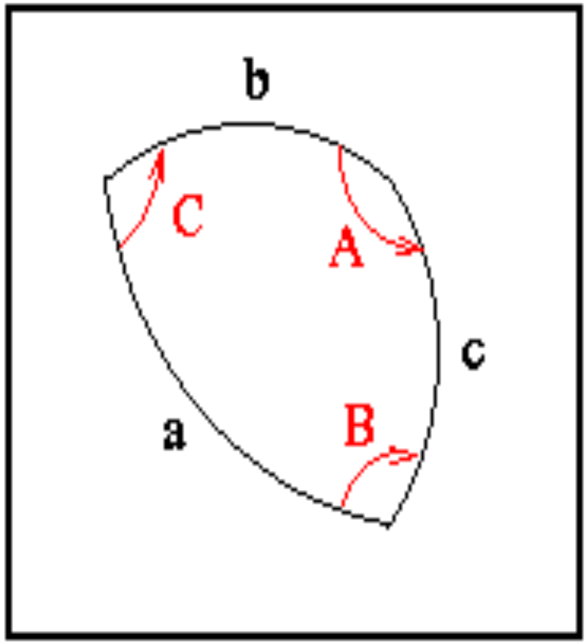
$$\frac{\operatorname{sen} a}{\operatorname{sen} A} = \frac{\operatorname{sen} b}{\operatorname{sen} B} = \frac{\operatorname{sen} c}{\operatorname{sen} C}$$

Seno & Co-seno

$$\operatorname{sen} a \,.\, \cos B = \cos b \,.\, \operatorname{sen} c - \operatorname{sen} b \,.\, \cos c \,.\, \cos A$$

- ❑ Lei dos cossenos para os lados (três lados e um ângulo):

$$\cos a = \cos b \cos c + \text{sen}\, b\, \text{sen}\, c \cos A$$
$$\cos b = \cos a \cos c + \text{sen}\, a\, \text{sen}\, c \cos B$$
$$\cos c = \cos a \cos b + \text{sen}\, a\, \text{sen}\, b \cos C$$



- ❑ Lei dos cossenos para os ângulos (três ângulos e um lado):

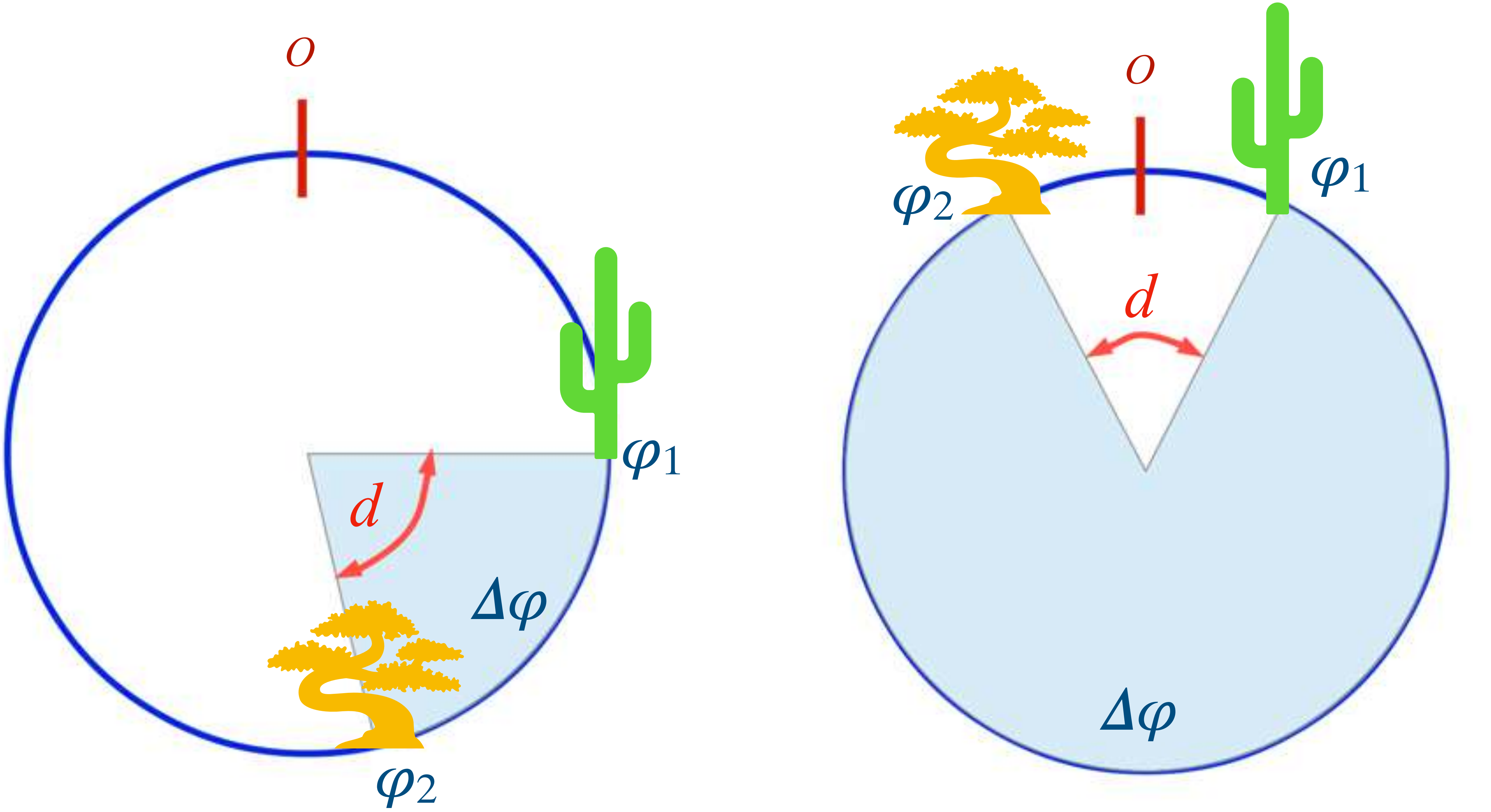
$$\cos A = - \cos B \cos C + \text{sen}\, B\, \text{sen}\, C \cos a$$
$$\cos B = - \cos A \cos C + \text{sen}\, A\, \text{sen}\, C \cos b$$
$$\cos C = - \cos A \cos B + \text{sen}\, A\, \text{sen}\, B \cos c$$

- ❑ Lei dos senos (dois lados e dois ângulos respectivamente opostos do triângulo esférico):

$$\text{sen}\, a / \text{sen}\, A = \text{sen}\, b / \text{sen}\, B = \text{sen}\, c / \text{sen}\, C$$

$\Delta\varphi$ separação angular (medida em relação a uma origem comum)

d “distância" angular (menor separação entre os objetos)

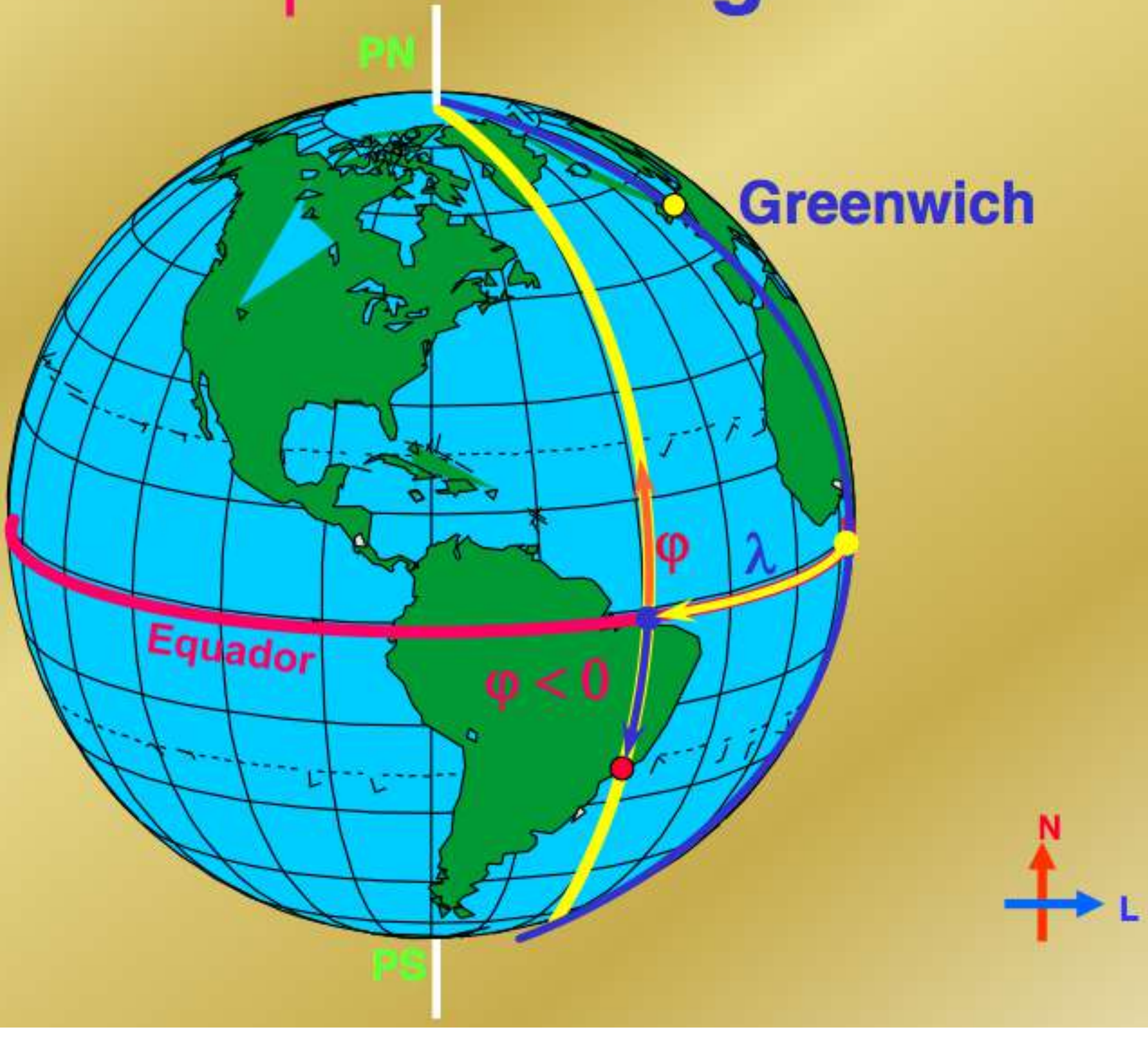


Se $|\Delta\varphi| \leq 180^\circ \rightarrow d = \Delta\varphi$

Se $|\Delta\varphi| > 180^\circ \rightarrow d = 360^\circ - \Delta\varphi$

Esfera Terrestre: ângulo entre duas cidades

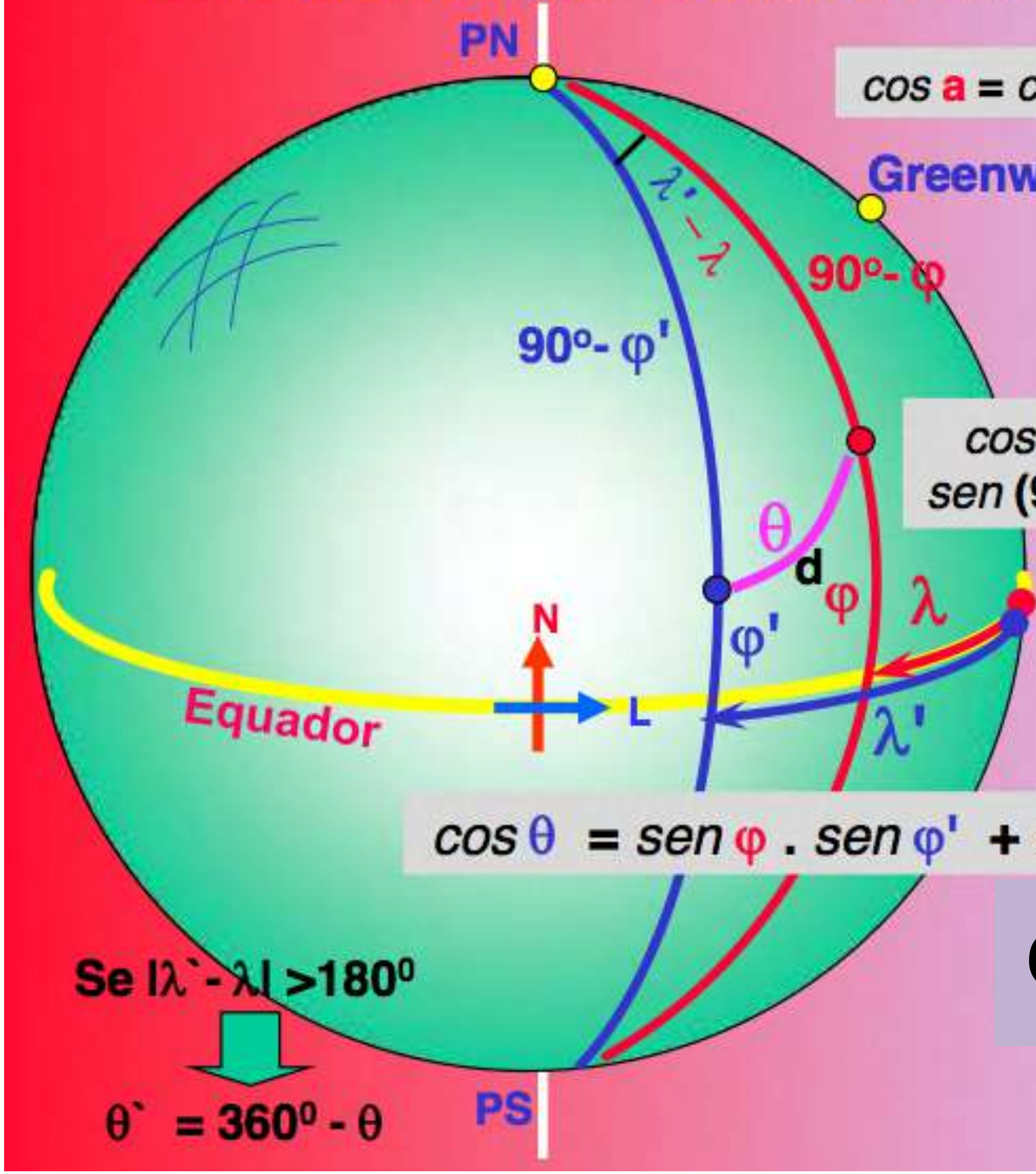
Latitude φ e Longitude λ



Distância entre duas cidades

$\cos a = \cos b \, . \, \cos c + \operatorname{sen} b \, . \, \operatorname{sen} c \, . \, \cos A$

Co-seno



$$\cos \theta = \cos (90- \varphi) \, . \, \cos (90- \varphi') + \operatorname{sen} (90- \varphi) \, . \, \operatorname{sen} (90- \varphi') \, . \, \cos (\lambda' - \lambda)$$

$$\cos \theta = \operatorname{sen} \varphi \, . \, \operatorname{sen} \varphi' + \cos \varphi \, . \, \cos \varphi' \, . \, \cos (\lambda' - \lambda)$$

distância "d"

Se $|\lambda` - \lambda| > 180^0$

$\theta` = 360^0 - \theta$

Ângulo entre dois astros no Sistema Altazimutal

Coordenadas Altazimutais

A = azimute

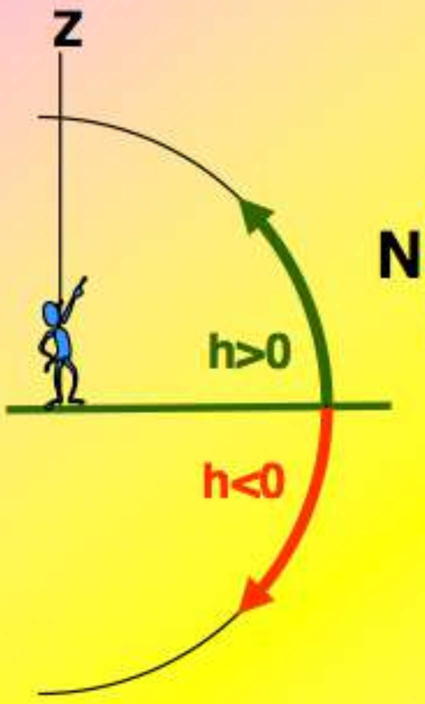
h = altura

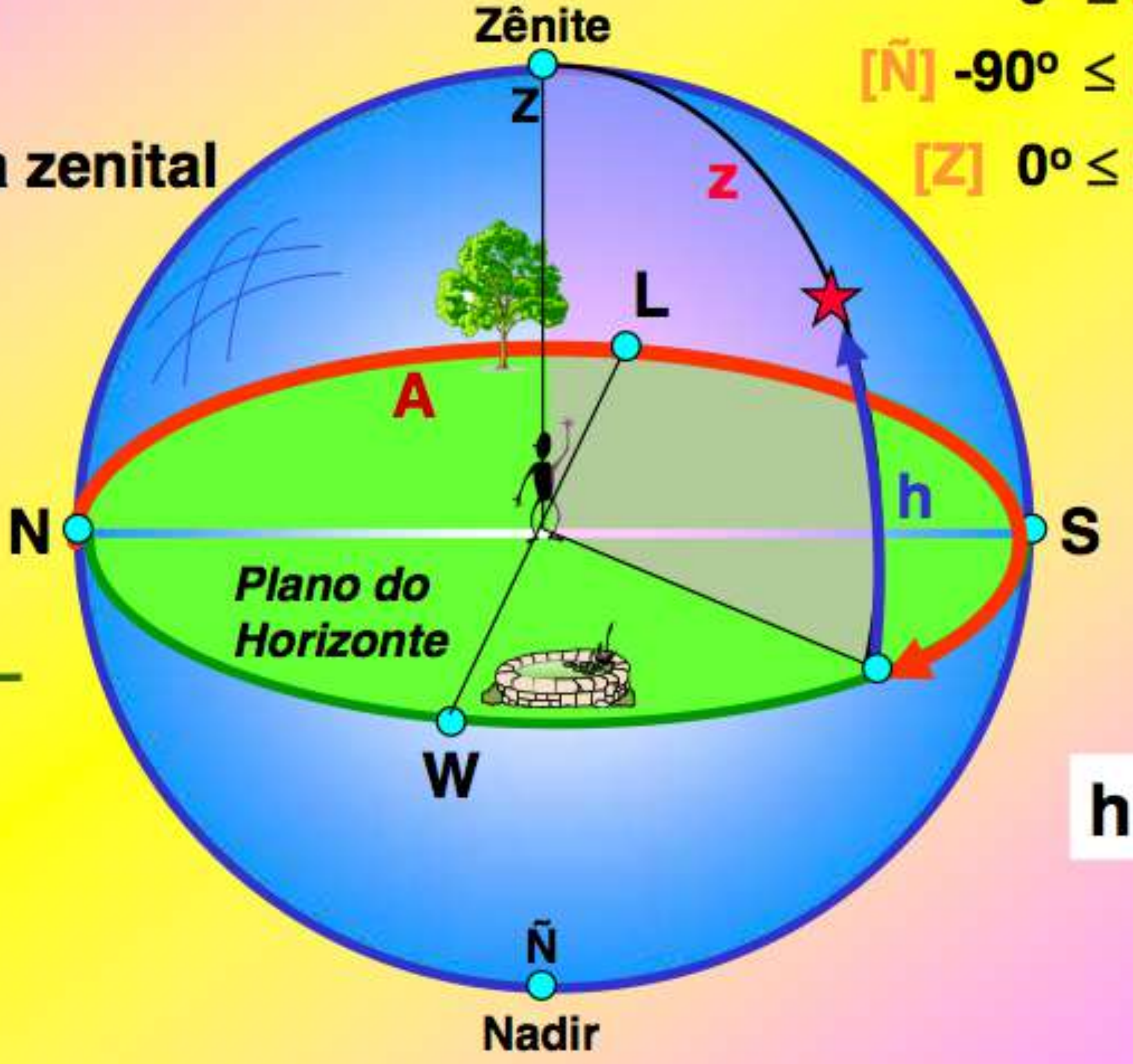
z = distância zenital

$0^o \le A < 360^o$

[Ñ] $-90^o \le h \le +90^o$ [Z]

[Z] $0^o \le z \le 180^o$ [Ñ]





★ A,h

★ A,z

$h + z = 90^0$

Ângulo entre dois astros

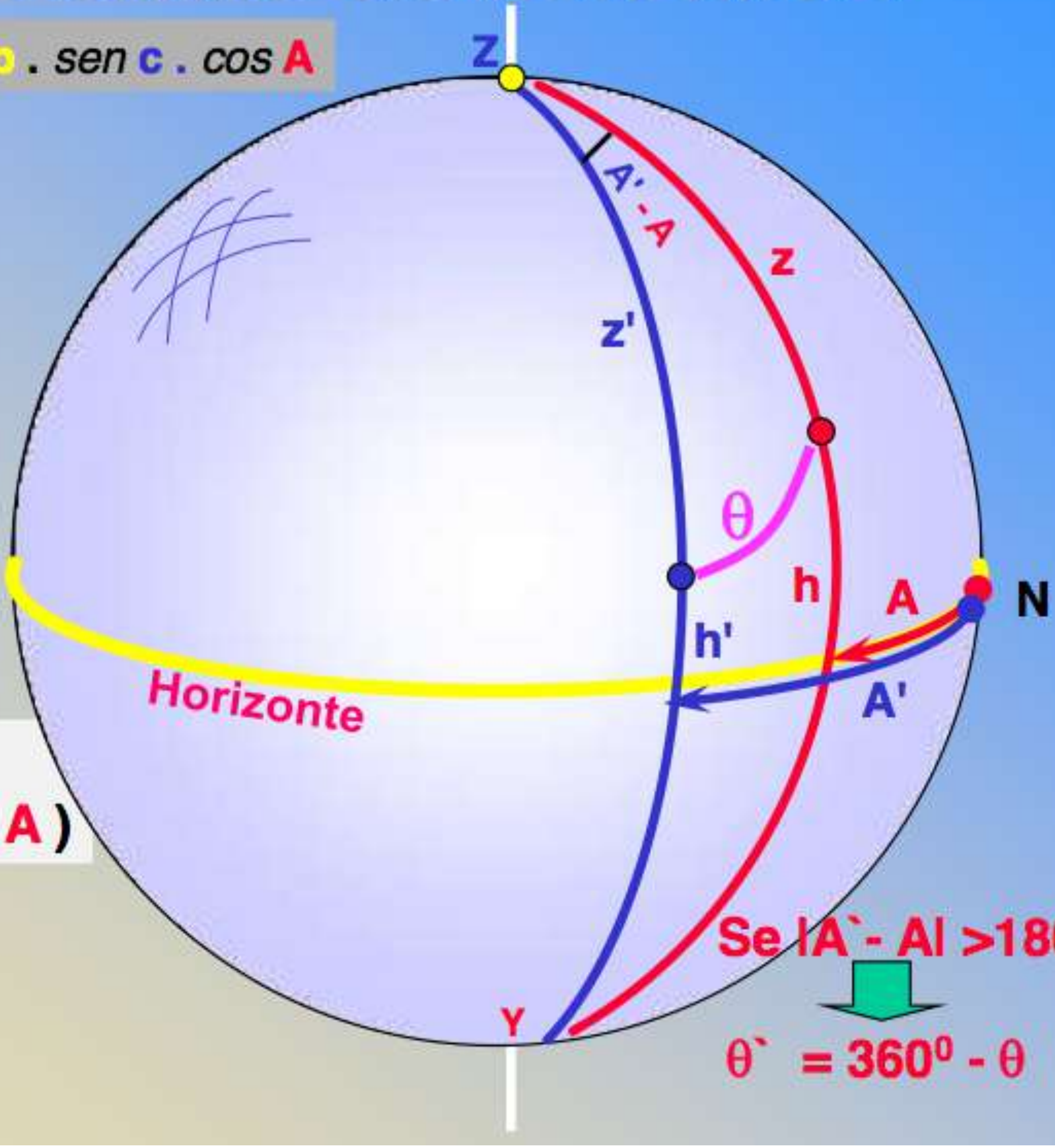
$$\cos a = \cos b\ .\ \cos c + \operatorname{sen} b\ .\ \operatorname{sen} c\ .\ \cos A$$

Co-seno

Dados:

A , z

A' , z'



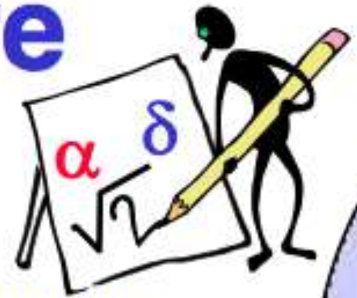
$$\cos \theta = \cos z\ .\ \cos z' + \operatorname{sen} z\ .\ \operatorname{sen} z'\ .\ \cos\,(A' - A)$$

Se |A` - A| >180^0

⇓

$\theta` = 360^0 - \theta$

Ângulo entre 2 astros



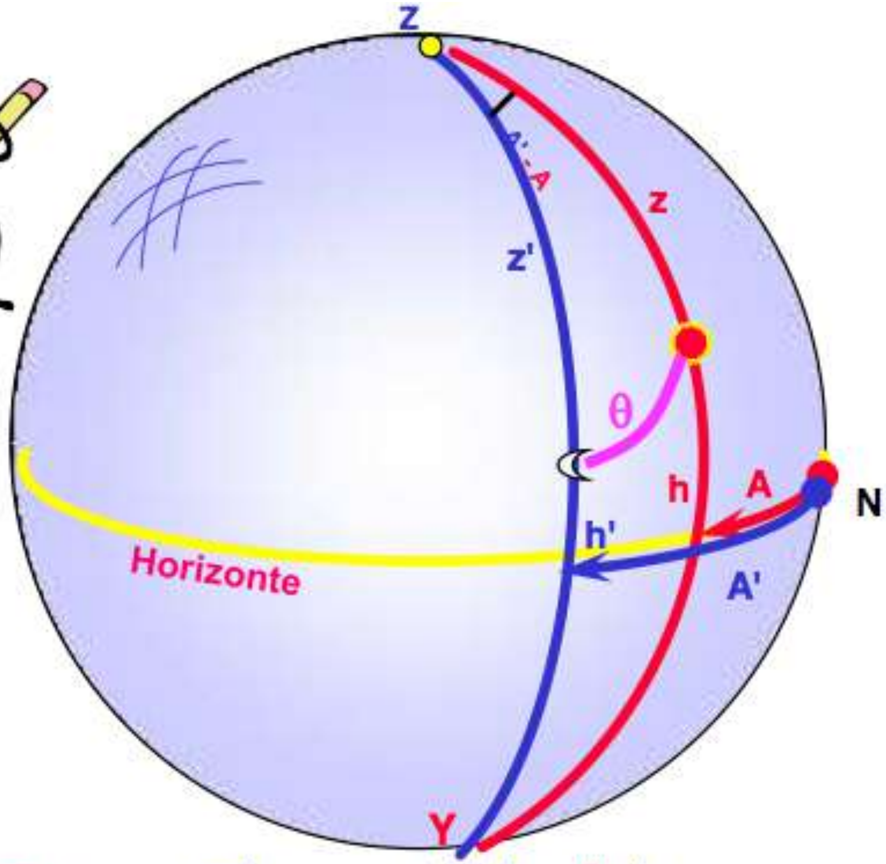
Dados:

● $A1 = 4^0\ 05'\ 06.000'' = 4.08500000^0$

$Z1 = 10^0\ 20'\ 30.000'' = 10.34166667^0$

☾ $A2 = 280^0\ 50'\ 30.000'' = 280.84166667^0$

$Z2 = 40^0\ 10'\ 10.000'' = 40.16944444^0$



$$\cos\theta = \cos z \,.\, \cos z' + \text{sen}\, z \,.\, \text{sen}\, z' \,.\, \cos(A' - A)$$

cos θ =cosZ1 * cosZ2 + sinZ1 * sinZ2 * cos(A2-A1)

cos θ = 0.98375475* 0.76414014+ 0.17951767* 0.64505027* 0.11765295

cos θ = 0.76535046

$\theta = 40^0\ 03'\ 42.551'' = 40.06181965^0$

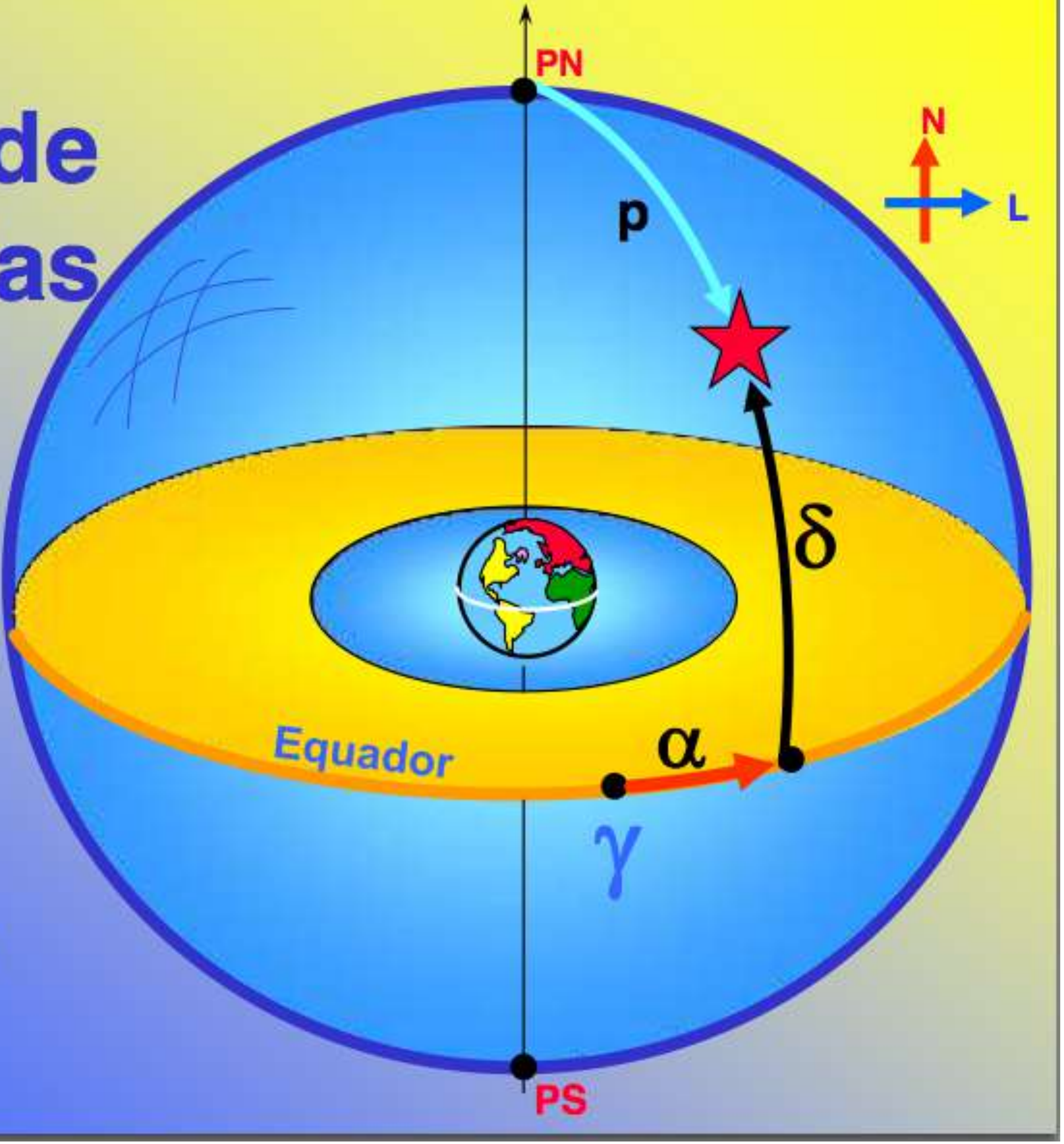
Como $|A_2 - A_1| > 180^0$

$\theta` = 360^0 - \theta$

$\theta` = 319^0\ 56'\ 17{,}449''$

Ângulo entre dois astros no Sistema Equatorial

Sistema Equatorial de Coordenadas



$p + \delta = 90^0$

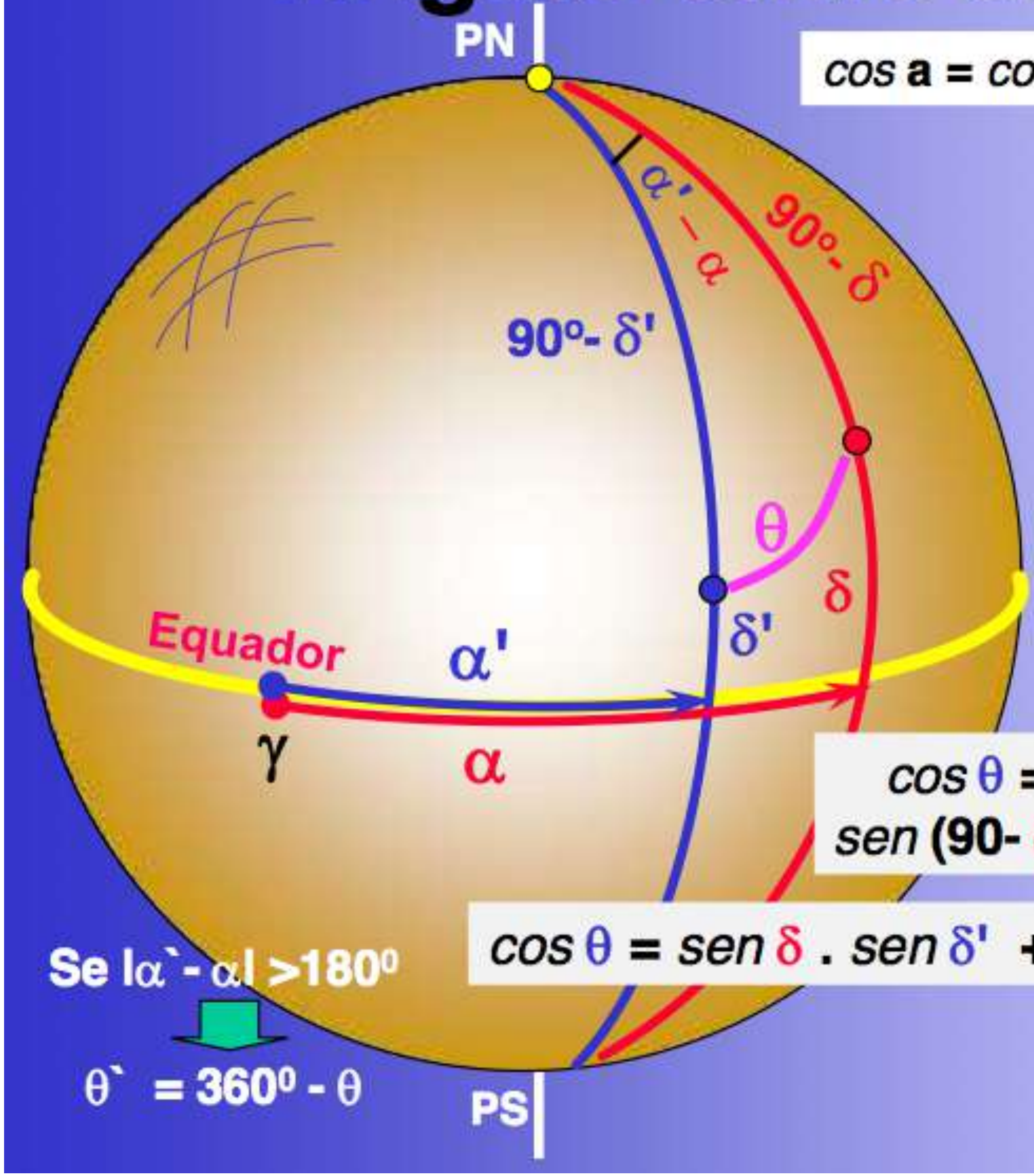
★ (α, δ)

α = ascensão reta

δ = declinação

p = distância polar

Ângulo entre dois astros



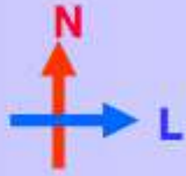
$cos\,\mathbf{a} = cos\,\mathbf{b}\,.\,cos\,\mathbf{c} + sen\,\mathbf{b}\,.\,sen\,\mathbf{c}\,.\,cos\,\mathbf{A}$

Co-seno

Dados:

$\alpha\ ,\ \delta$

$\alpha'\ ,\ \delta'$



$$cos\,\theta = cos\,(90-\delta)\,.\,cos\,(90-\delta')\,+$$
$$sen\,(90-\delta)\,.\,sen\,(90-\delta')\,.\,cos\,(\alpha'-\alpha)$$

$$cos\,\theta = sen\,\delta\,.\,sen\,\delta' + cos\,\delta\,.\,cos\,\delta'\,.\,cos\,(\alpha'-\alpha)$$

Se $|\alpha` - \alpha| > 180^0$

⇩

$\theta` = 360^0 - \theta$

Ângulo entre 2 astros

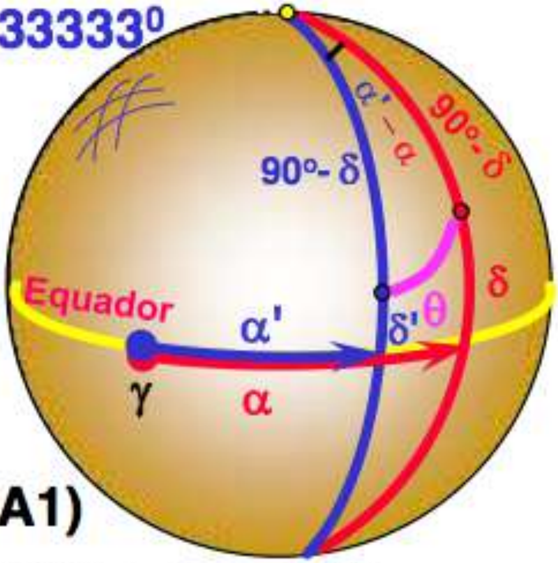
Dados

Alfa1 = 3h 10m 50.000s = 3.18055556 h = 47.70833333^0

Delta1 = 20^0 30' 40.000" = 20.51111111^0

Alfa2 = 5h 30m 20.000s = 5.50555556 h = 82.58333333^0

Delta2 = -10^0 15' 20.000" = -10.25555556^0



$$\cos\theta = \operatorname{sen}\delta \,.\, \operatorname{sen}\delta' + \cos\delta \,.\, \cos\delta' \,.\, \cos(\alpha' - \alpha)$$

cosTheta=sinD1 * sinD2 + cosD1 * cosD2 * cos(A2-A1)

cosTheta= 0.35038902*-0.17803896+ 0.93660426* 0.98402344* 0.82040144

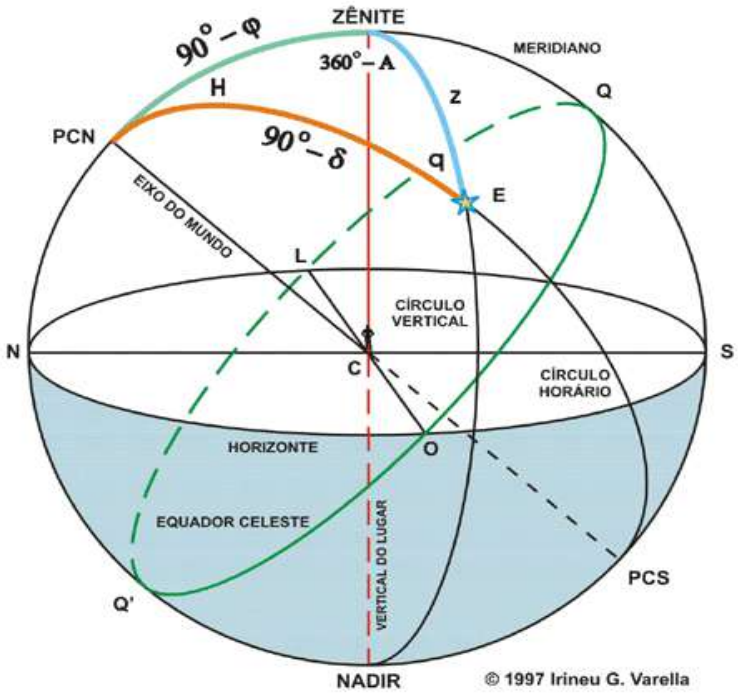
cosTheta= 0.69373233

Theta = 46^0 04' 25.368" = 46.07371342^0

Como $|\alpha_2 - \alpha_1| < 180^0$

$\Downarrow$

$\theta = 46^0\ 04'\ 25.368''$



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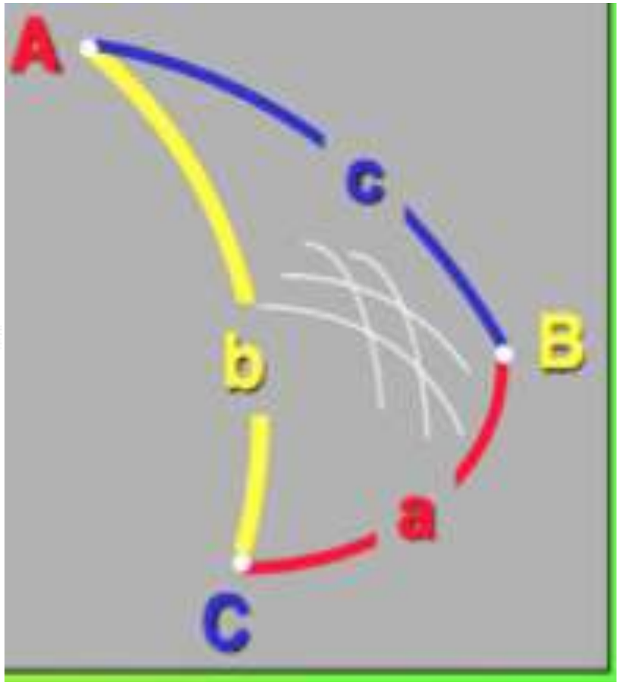


Triângulo: PEZ

$A = H$ $a = z$

$B = 360 – A$ $b = 90 – \delta$

$C = q$ $c = 90 – \Phi$

q: ângulo paralático

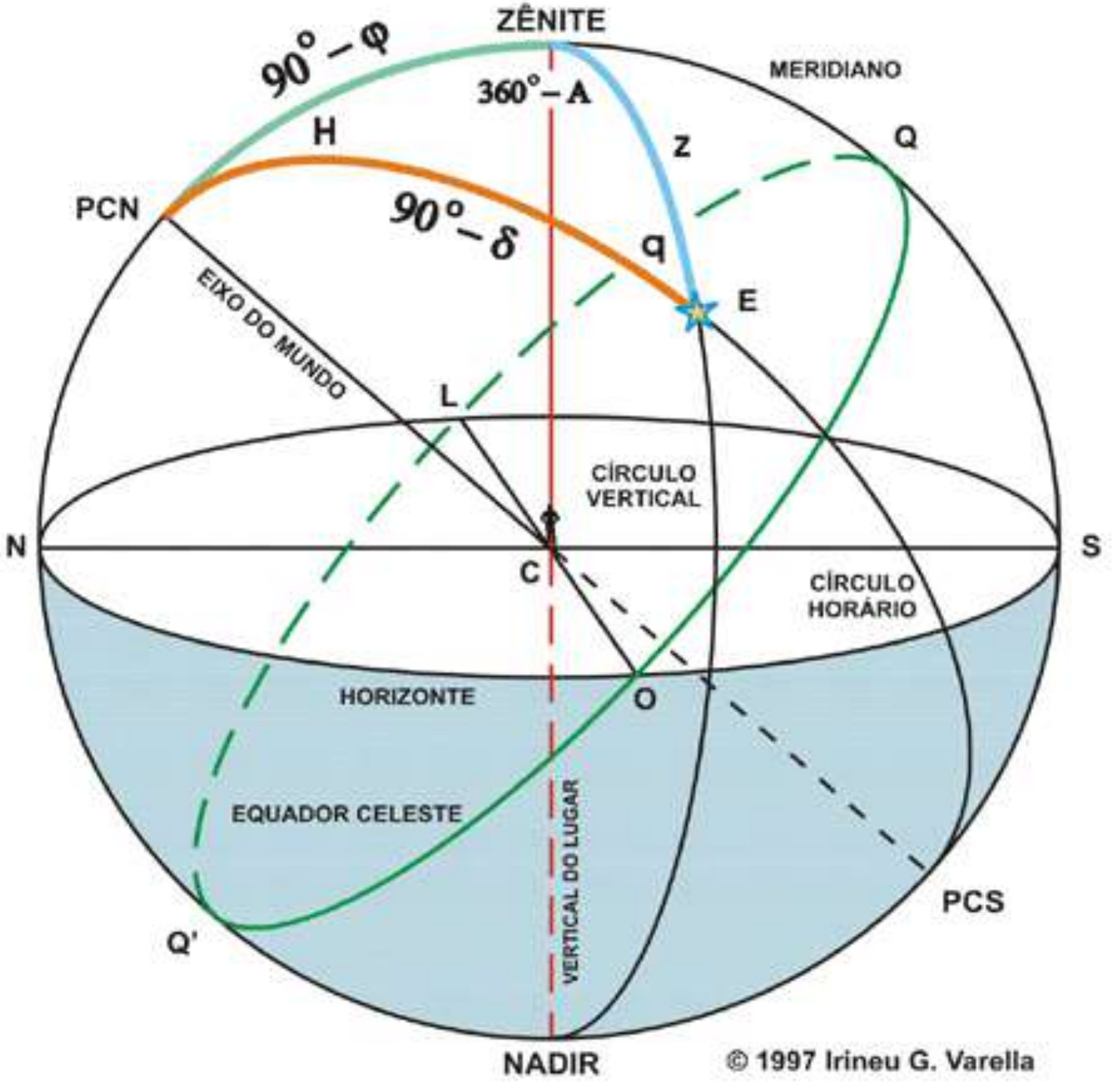
Leis do co-seno

$$\cos(90 - \delta) = \cos(z).\cos(90 - \Phi) + \text{sen}(z).\text{sen}(90 - \Phi).\cos(360\text{-}A)$$

O triângulo de posição

Importância para a Astronomia

Em qualquer instante, exceto pela passagem meridiana, um astro forma um triângulo esférico com o pólo celeste de seu hemisfério equatorial e com o zênite do observador.



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Relação entre as coordenadas horárias e horizontais

$$\mathrm{sen}(\delta) = \cos(z).\mathrm{sen}(\Phi) + \mathrm{sen}(z).\cos(\Phi).\cos(A)$$

Aplicando a Lei dos Senos

$$\mathrm{sen}(360-A)/[\mathrm{sen}(90-\delta)] = \mathrm{sen}(H)/\mathrm{sen}(z)$$

Aplicando a Lei dos Senos

$$\mathrm{sen}(H).\cos(\delta) = -\mathrm{sen}(z).\mathrm{sen}(A)$$

Aplicando a Lei dos Senos/Cos-senos por conta do sinal de H

$$\mathrm{sen}(90 -\delta).\cos(H) = \cos(z).\mathrm{sen}(90 - \Phi) - \mathrm{sen}(z).\cos(90 - \Phi).\cos(360-A)$$

Aplicando a Lei dos Senos/Cos-senos por conta do sinal de H

$$\cos(\delta).\cos(H) = \cos(z).\cos(\Phi) - \text{sen}(z).\text{sen}(\Phi).\cos(A)$$

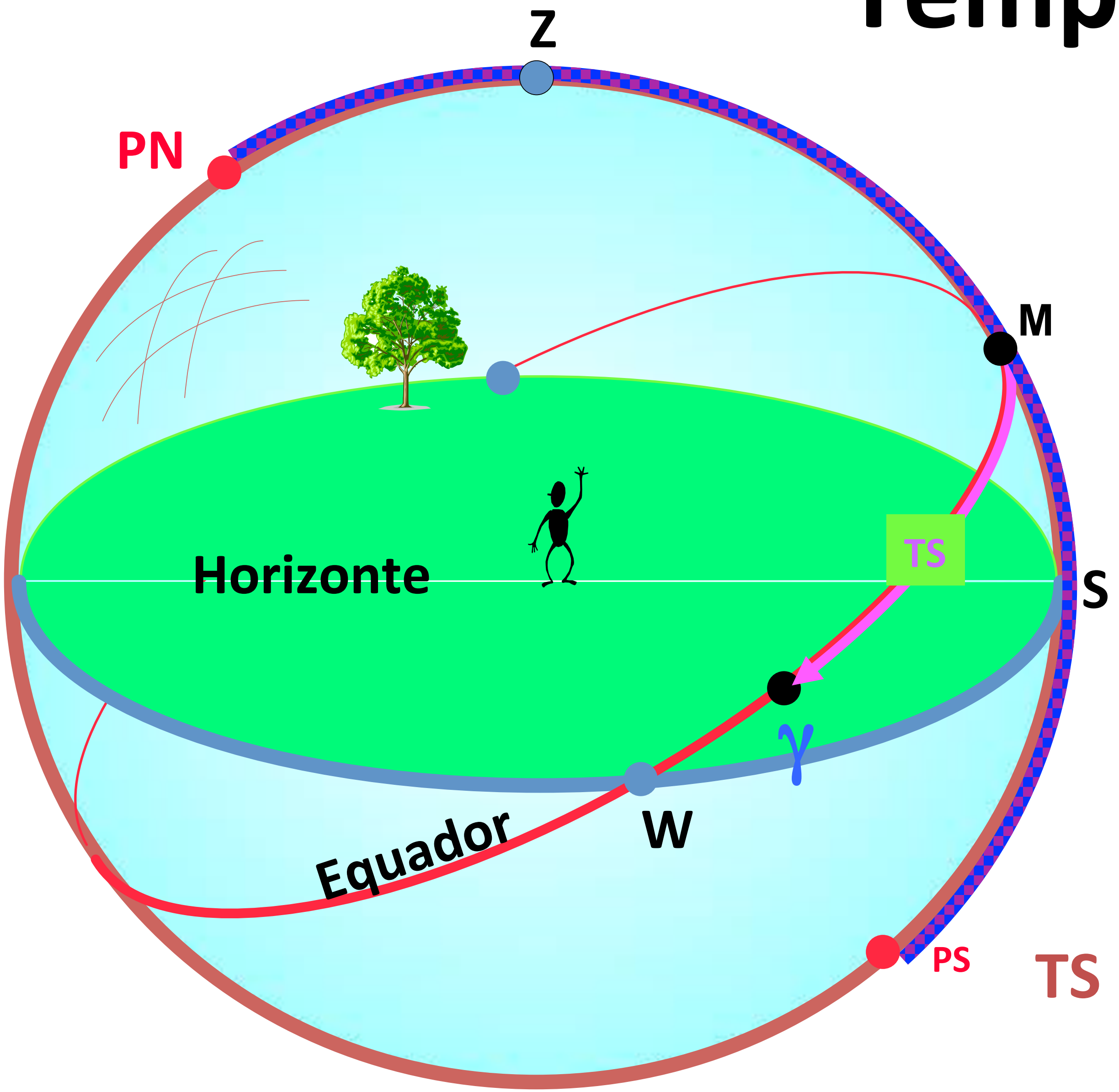
Coordenadas Equatoriais em função das Horárias

$$\cos(z) = \text{sen}(\Phi).\text{sen}(\delta) + \cos(\Phi).\cos(\delta).\cos(H)$$

$$\text{sen}(z).\text{sen}(A) = -\text{sen}(H).\cos(\delta)$$

$$\text{sen}(z).\cos(A) = \cos(\Phi).\text{sen}(\delta) - \text{sen}(\Phi).\cos(\delta).\cos(H)$$

Tempo sideral



Tempo sideral é o ângulo horário do ponto γ

$$TS = H_{\gamma} = H_{*} + \alpha_{*}$$

$H = T - \alpha$

Já vimos que:

$$\text{sen}(\delta) = \cos(z).\text{sen}(\Phi) + \text{sen}(z).\cos(\Phi).\cos(A)$$

Substituindo $H = T - \alpha$ nas equações anteriores:

$$\text{sen}(T - \alpha).\cos(\delta) = -\text{sen}(z).\text{sen}(A)$$

$$\cos(\delta).\cos(T - \alpha) = \cos(z).\cos(\Phi) - \text{sen}(z).\text{sen}(\Phi).\cos(A)$$

$$\cos(z) = \text{sen}(\Phi).\text{sen}(\delta) + \cos(\Phi).\cos(\delta).\cos(T - \alpha)$$

$$\text{sen}(z).\text{sen}(A) = -\text{sen}(T - \alpha).\cos(\delta)$$

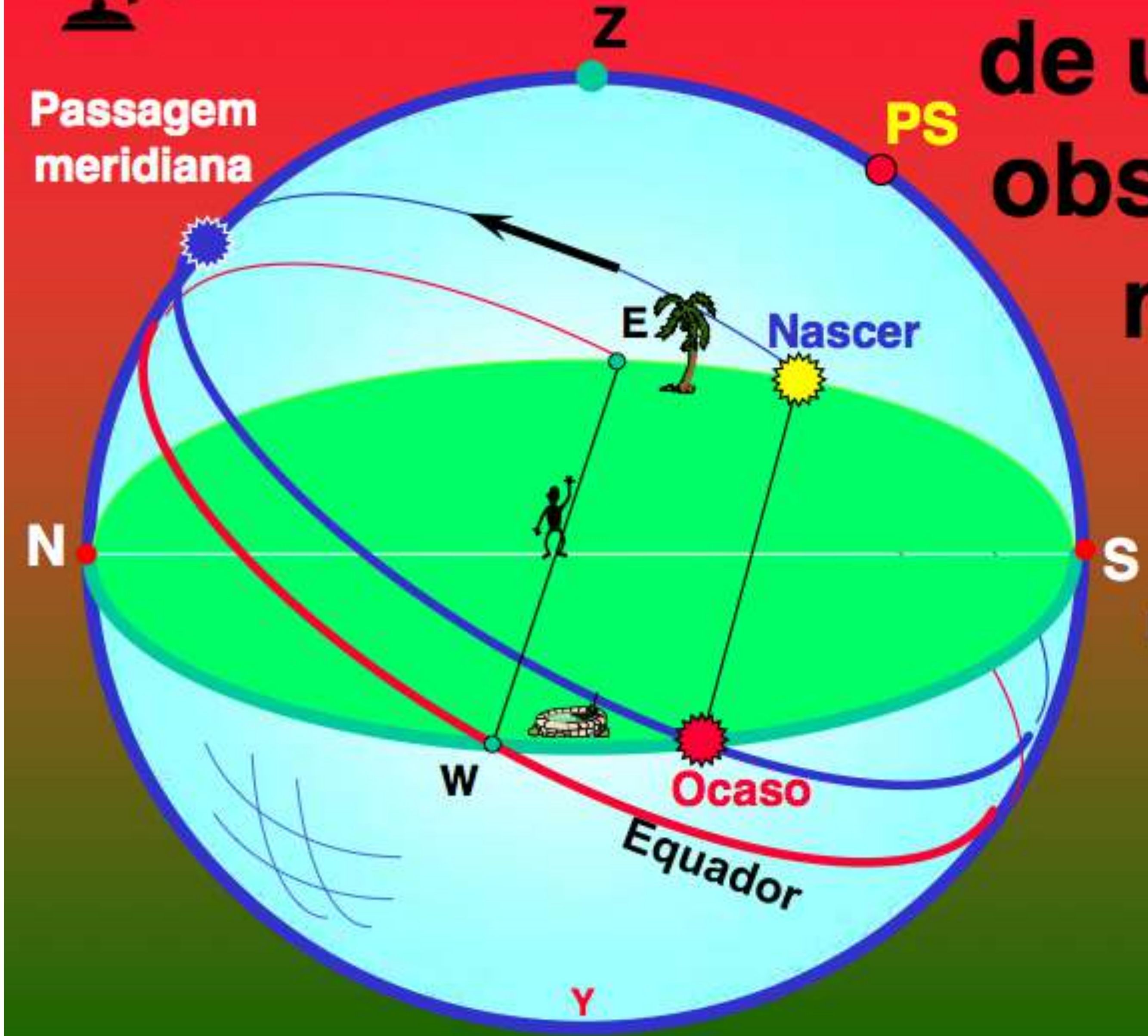
$$\text{sen}(z).\cos(A) = \cos(\Phi).\text{sen}(\delta) - \text{sen}(\Phi).\cos(\delta).\cos(T - \alpha)$$

3 primeiras: Sistema Horizontal --- Equatorial

3 últimas: Sistema Equatorial --- Local Horizontal

Nascer e ocaso

Nascer e Ocaso de um astro observador no HS



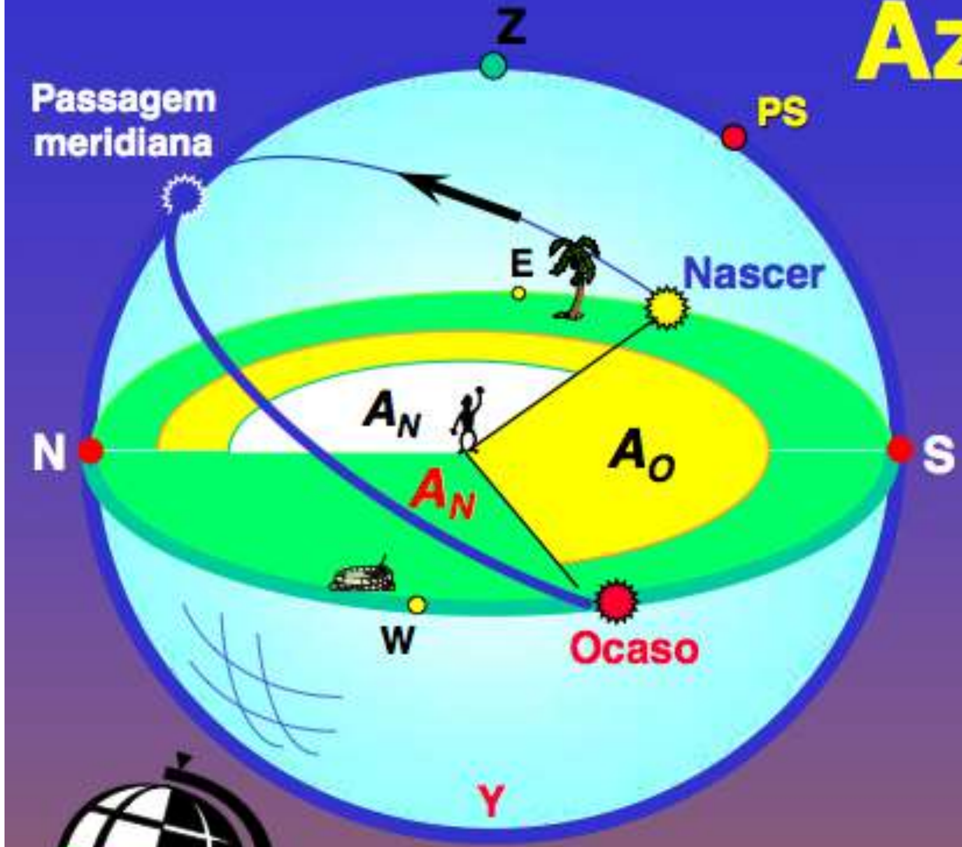
Condição de nascer e de ocaso:

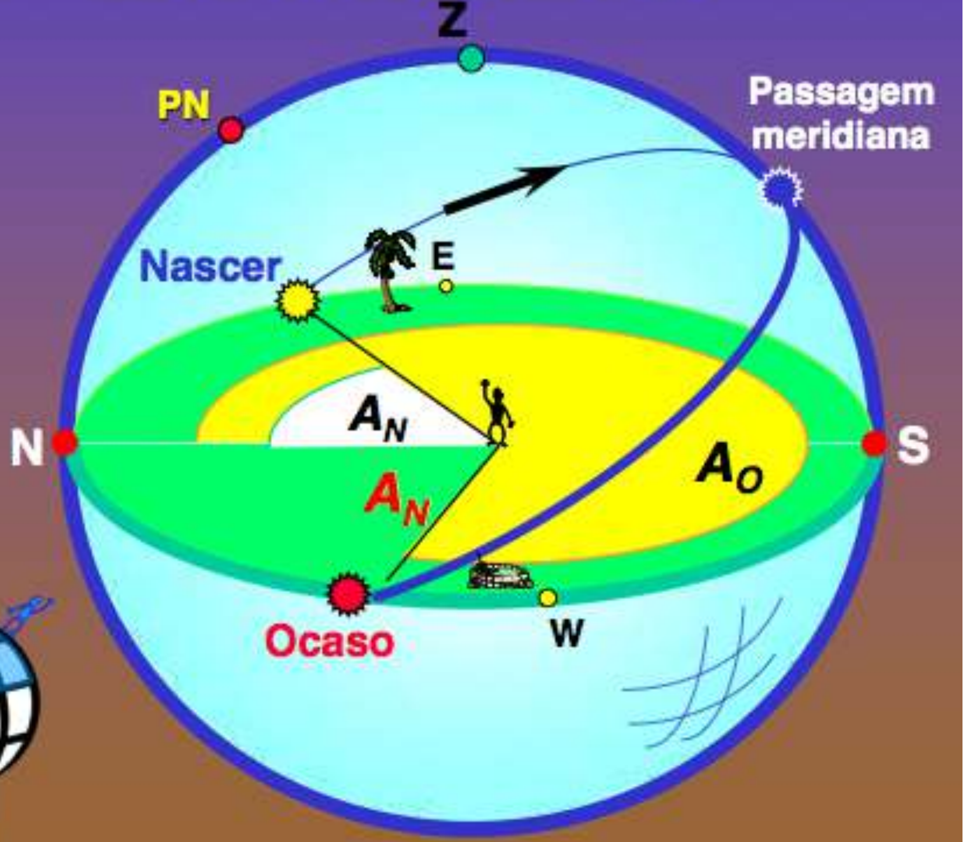
h = 0

ou

z = 90º

Azimutes do nascer e do ocaso nos diferentes hemisférios



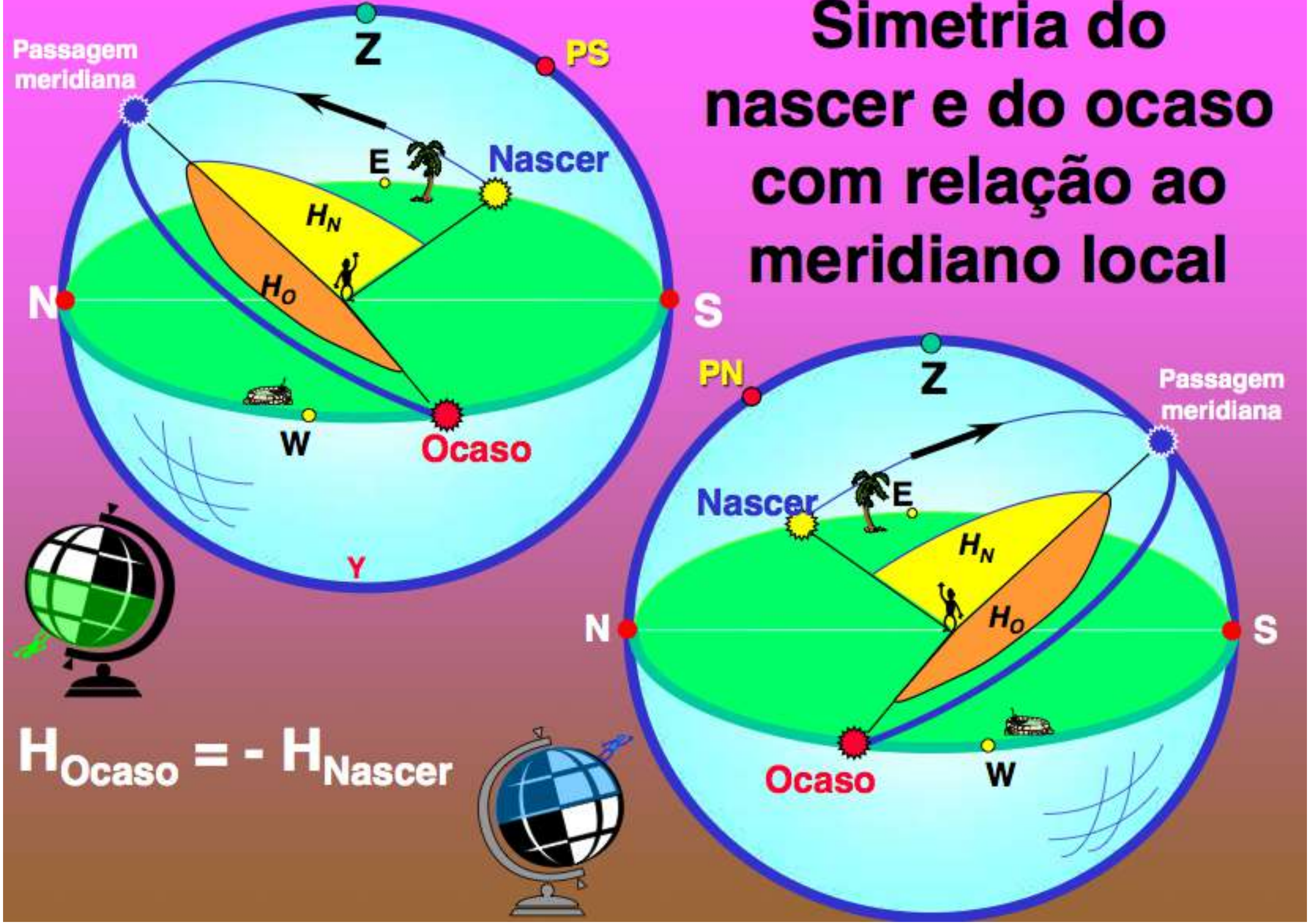


A_N: Azimute Nascer

A_O : Azimute Ocaso

$$A_{Ocaso} = 360^o - A_{Nascer}$$

Simetria do nascer e do ocaso com relação ao meridiano local

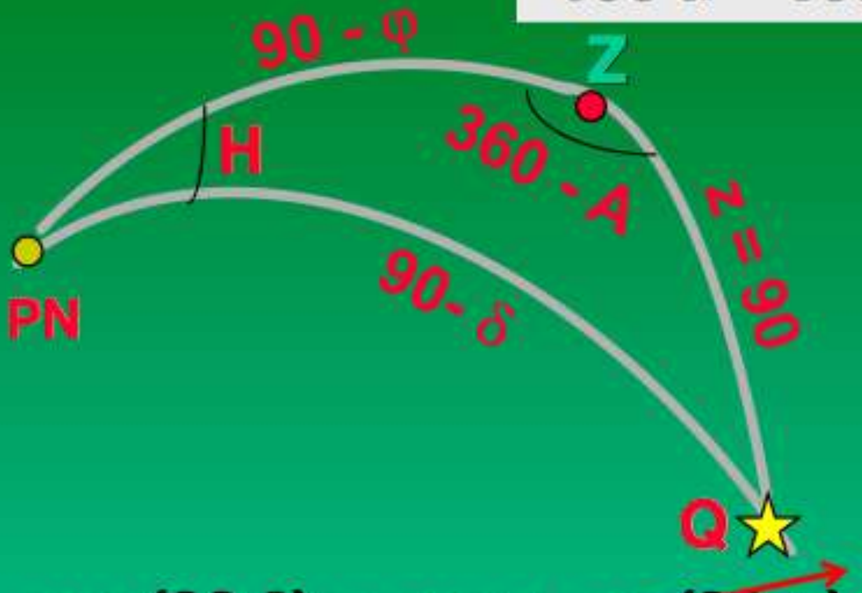


$$H_{Ocaso} = - H_{Nascer}$$

Triângulo esférico no ocaso de um astro

Azimute do Nascer e do Ocaso

$$\cos \mathbf{a} = \cos \mathbf{b} \cdot \cos \mathbf{c} + \mathrm{sen}\, \mathbf{b} \cdot \mathrm{sen}\, \mathbf{c} \cdot \cos \mathbf{A}$$



$\mathbf{a} = 90 - \delta$
$\mathbf{b} = z = 90$ **(distância zenital)**
$\mathbf{c} = 90 - \varphi$
$\mathbf{A} = 360 - A$

$$\cos(90-\delta) = \cancel{\cos z \cdot \cos}(90-\varphi) + \mathrm{sen}\,(90-\varphi) \cdot \overset{= 1.0}{\cancel{\mathrm{sen}\, z}} \cdot \cos(360-A)$$

$$\cos(90-\delta) = \mathrm{sen}\,(90-\varphi) \cdot \cos(360-A)$$

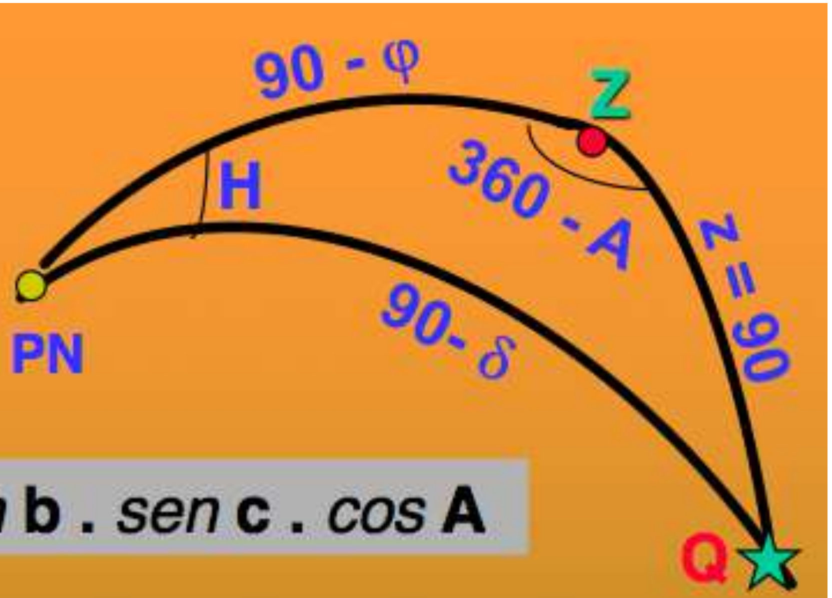
$$\mathrm{sen}\,\delta = \cos\varphi \cdot \cos A$$

$$\boxed{\cos \mathbf{A} = \mathrm{sen}\,\delta / \cos\varphi}$$ 🟥 $\Rightarrow 0 \leq \underline{A} \leq 180^o$

No Nascer: $A = \underline{A}$

No Ocaso: $A = 360^o - \underline{A}$

Ângulo horário no nascer e no ocaso



$$\cos \mathbf{a} = \cos \mathbf{b} \,.\, \cos \mathbf{c} + \operatorname{sen} \mathbf{b} \,.\, \operatorname{sen} \mathbf{c} \,.\, \cos \mathbf{A}$$

$$\cos \mathbf{z} = \cos (90\text{-}\varphi) \,.\, \cos (90\text{-}\delta) + \operatorname{sen} (90\text{-}\varphi) \,.\, \operatorname{sen} (90\text{-}\delta) \,.\, \cos \mathrm{H}$$

$$0 = \operatorname{sen} \varphi \,.\, \operatorname{sen} \delta + \cos \varphi \,.\, \cos \delta \,.\, \cos \mathrm{H}$$

$$\cos \mathrm{H} = - \operatorname{sen} \varphi \,.\, \operatorname{sen} \delta \,/\, \cos \varphi \,.\, \cos \delta$$

$$\boxed{\cos \mathrm{H} = - \tan \varphi \,.\, \tan \delta}$$

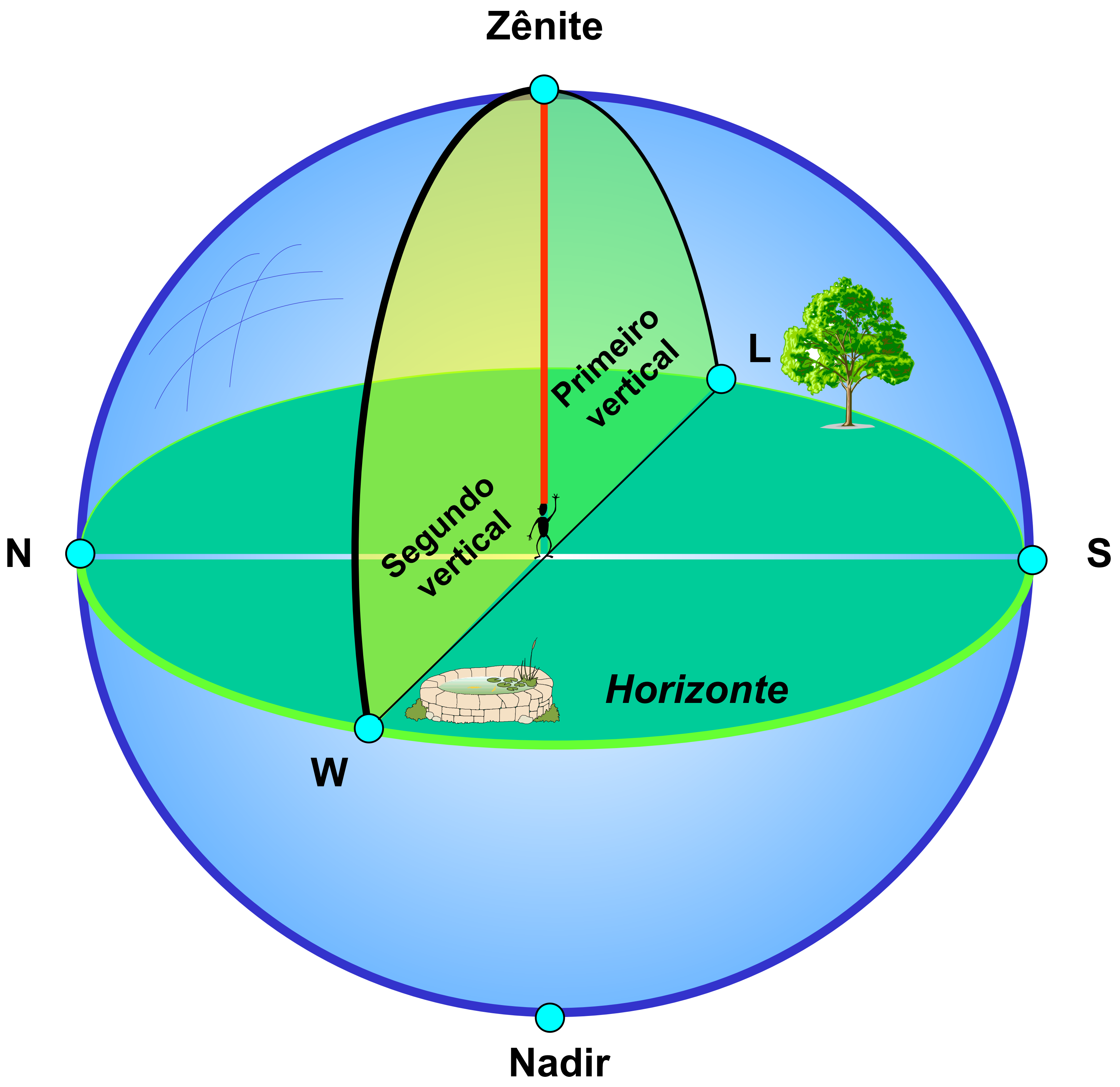
🟥 $\Rightarrow 0 \leq \underline{\mathrm{H}} \leq 180^0$

No Ocaso: $\mathrm{H}_o = \underline{\mathrm{H}}$

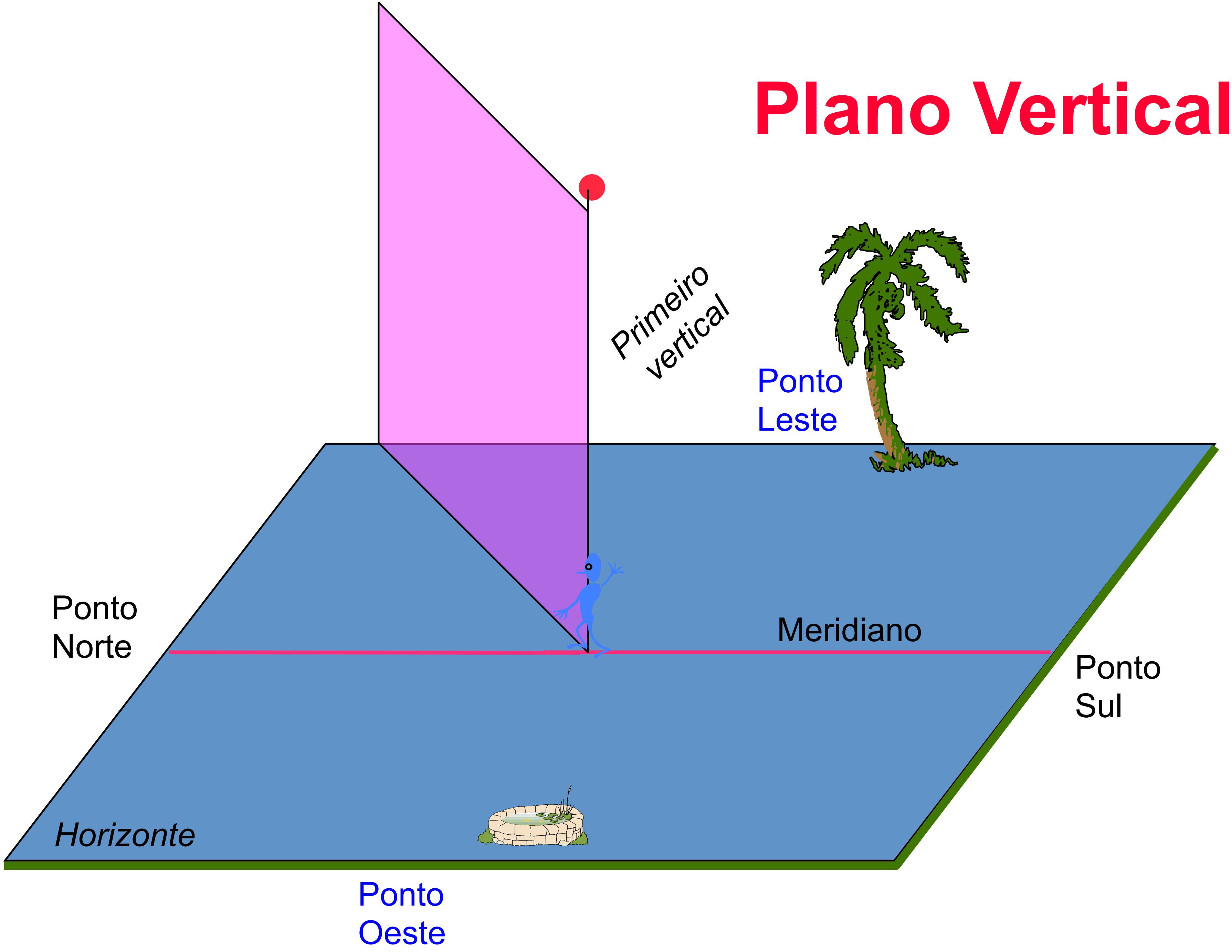
No Nascer: $\mathrm{H}_n = - \underline{\mathrm{H}}$

Cruzamento com o primeiro e segundo verticais

Primeiro e segundo verticais



Plano Vertical



$$\mathrm{sen}(\delta) = \cos(z).\mathrm{sen}(\Phi) + \mathrm{sen}(z).\cos(\Phi).\cos(A)$$

$$\mathrm{sen}(H).\cos(\delta) = -\mathrm{sen}(z).\mathrm{sen}(A)$$

$$\cos(\delta).\cos(H) = \cos(z).\cos(\Phi) - \mathrm{sen}(z).\mathrm{sen}(\Phi).\cos(A)$$
$$\mathrm{sen}(z).\cos(A) = \cos(\Phi).\mathrm{sen}(\delta) - \mathrm{sen}(\Phi).\cos(\delta).\cos(H)$$

$A= 90^0$ (Primeiro Vertical)

$A = 270^0$ (Segundo Vertical)

$\delta <= \Phi$ (N) ou $\delta >= \Phi$ (S)

$$\mathrm{sen}(\delta) = \cos(z).\mathrm{sen}(\Phi)$$
$$\mathrm{sen}(z) = +/- \mathrm{sen}(H).\cos(\delta)$$
$$\cos(\delta).\cos(H) = \cos(z).\cos(\Phi)$$
$$\cos(H) = \cot(\Phi).\tan(\delta)$$